STAT758

Final Project

Time series analysis of daily exchange rate between the British Pound and the US dollar (GBP/USD)

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INTRODUCTION

Time Series Analysis accounts for the fact that data points taken over time may have an internal structure (such as autocorrelation, trend or seasonal variation) that should be accounted for. The behavior of time series variables such as exchange rates is not consistent and to forecast it is irrational. Despite these assertions, many multinational corporations, dealers in foreign exchange, exporters, importers and speculators continue to make hedging decisions based on forecasted rates using ex-post data as their basis. These hedging decisions are made under the premise that patterns exist in the ex-post data and these patterns provide an indication of future movement of exchange rates, at least in the short run. If such patterns exist, then it is possible in principle to apply modern mathematical tools and techniques such as ARIMA and GARCH to forecast the ex-ante exchange rates (Hamilton 1994, Klaassen 1998).

The ARMA model is made up of two processes: the Autoregressive AR and the Moving Average MA. Given a series X_t, we can model the level of its current observations depends on the level of its lagged observations. This way of thinking can be represented by the AR model.

Also we can model that the observations of a random variable at time t are not only affected by the shock at time t, but also the shocks that have taken place before time t. This way of thinking can be represented by the MA model.

AutoRegressive Integrated Moving Average (ARIMA) models intend to describe the current behavior of variables in terms of linear relationships with their past values. It has an Integrated (I) component (d), which represents the amount of differencing to be performed on the series to make it stationary. The second component of the ARIMA consists of an ARMA model for the series rendered stationary through differentiation. The ARMA component is further decomposed into AR and MA components which are explained above. The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are used to estimate the values of the orders of the AR and MA processes respectively. The statistical package R is used to analyze the data.

OBJECTIVE

The goal of this study is to perform statistical analysis on the foreign exchange data between the GBP (Great Britain Pound) and the USD (United States dollar). The properties of the data are described and basic time series techniques are applied to the data. Plots of the series, autocorrelation function and the partial autocorrelation function are some of the graphical tools used to analyze the series. We also aim to fit a model (ARMA, ARCH, and GARCH) to the data in order to make credible forecasts from the model.

The data used for analysis is the close of business (COB) day value of the daily exchange rate between the British Pound and the US dollar (GBP/USD). The data was downloaded from the Oanda Corporation financial services website (http://www.oanda.com/currency/historical-rates/), from 1st January, 2000 to 4th April, 2012. Values on weekday holidays are assumed to be the value from the previous business day. For any other unexpected closure of the market of a single weekday, or several consecutive weekdays, the data points are filled in with the last COB value. A year of data is considered to be 52 weeks with five business days per week which equals 366 data points per year.

EXPLORATIVE DATA ANALYSIS AND DESCRIPTIVE OF STATISTICS

The data set is made up of 4478 daily quotes of foreign exchange rate between the GBP and the USD for the period year 2000 to 2012. The time series analysis for the exchange rate data is plotted using the logarithm of the returns of the rates. That is, if is the rate at time t, and is the rate at time t-1; then the logarithm of the returns is given as:

$$R_t = \log\left(\frac{p_t}{p_{t-1}}\right)$$

R_t = Return
P_t = Current rate
P_{t-1} = Previous day rate

This is because, the log returns of exchange rates have interesting statistical properties; thus making analysis easier. Also, the log returns are assumed to be normally distributed, which simplifies computations on the probabilistic aspect of the data. A histogram illustrating this assumption is shown in fig a below. Some basic (descriptive) statistics of this data set is given fig1. below.

Minimum	**1st Quartile**	**Median**	**Mean**	**3rd Quartile**	**Maximum**
-1.733e-02	-8.595e-04	0	-1.067e-06	8.822e-04	1.359e-02

Table 1. descriptive statistics of exchange rate of GBP/USD data

A histogram illustrating a bell shape, an indication of the normality assumption.

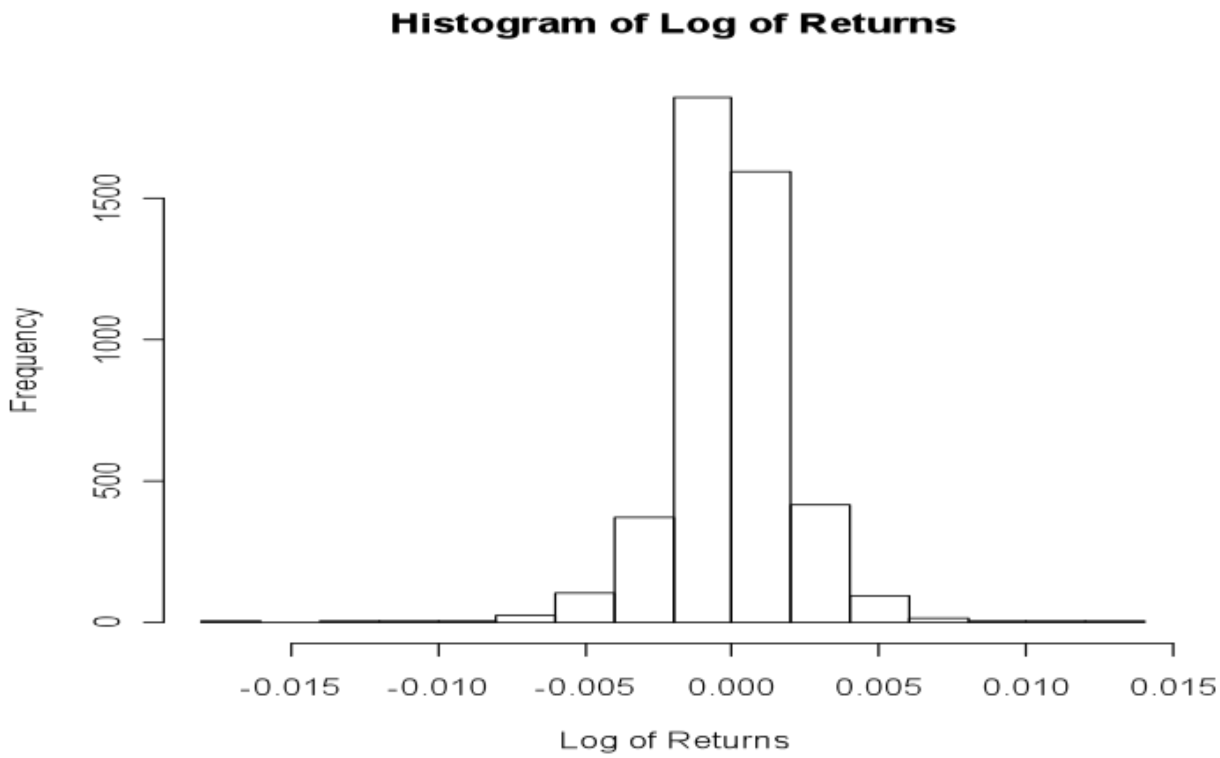

Fig 1. Histogram of the log return of the exchange rate of GBP/USD

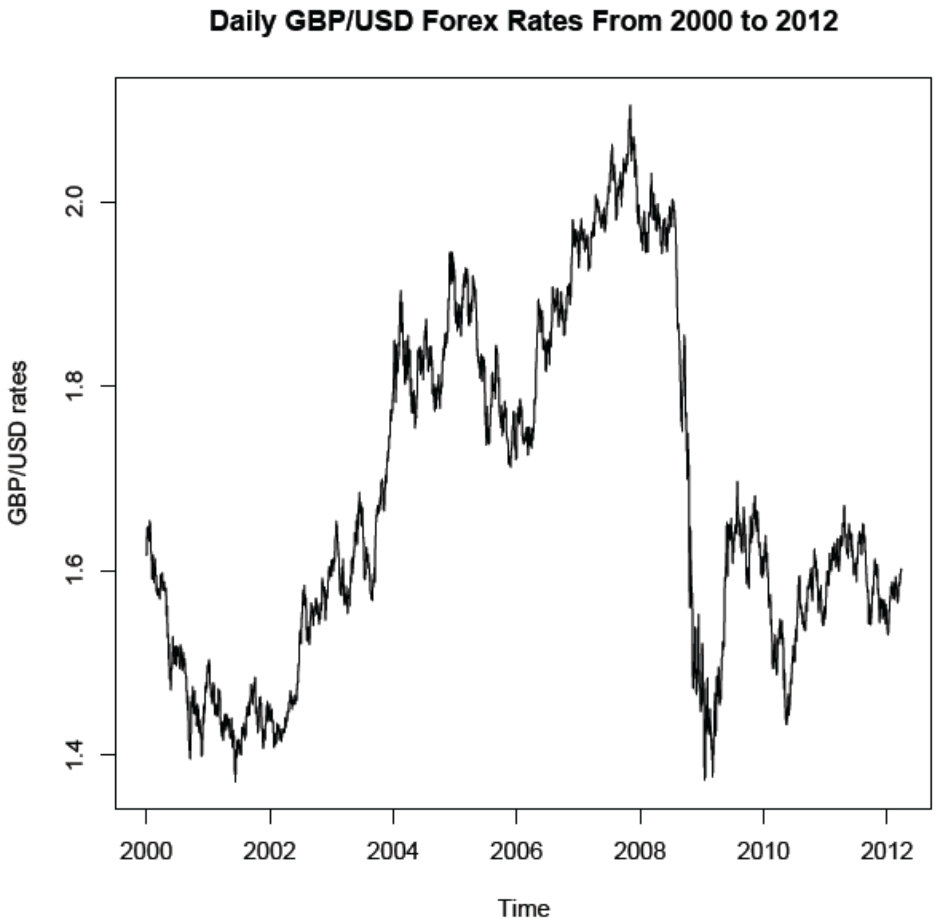

Fig 2. The plot of GBP/USD foreign exchange rates from 2000 to 2012

The time series plot for the returns is shown in fig. 2. It will be noted that this plot exhibits no periodicity but we do have a trend effect due to the random walk nature. After transformation using log returns, the trend effect is eliminated as seen in fig3. Thus, we do not have to apply any differencing technique to make the series stationary; therefore, the integration order is zero. Generally, the volatility of the series is fairly uniform for the years 2000 to 2006. For the years of 2006 to 2012, volatility of the series was highly non-uniform and more pronounced around 2009.

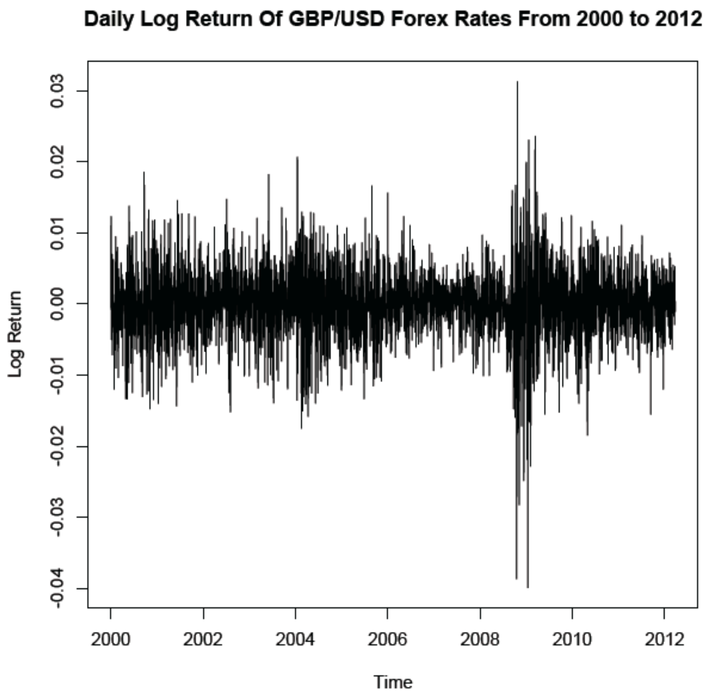

Fig 3. The plot of daily log return of GBP/USD foreign exchange rates from 2000 to 2012

One way to determine the order of the above series is to construct the ACF and PACF plots. The ACF plot is shown below in fig 3. with a two spikes out of the confidence range. ACF spikes for subsequent lags decay faster to zero with none being significant after lag one. This implies that the series has an MA order.

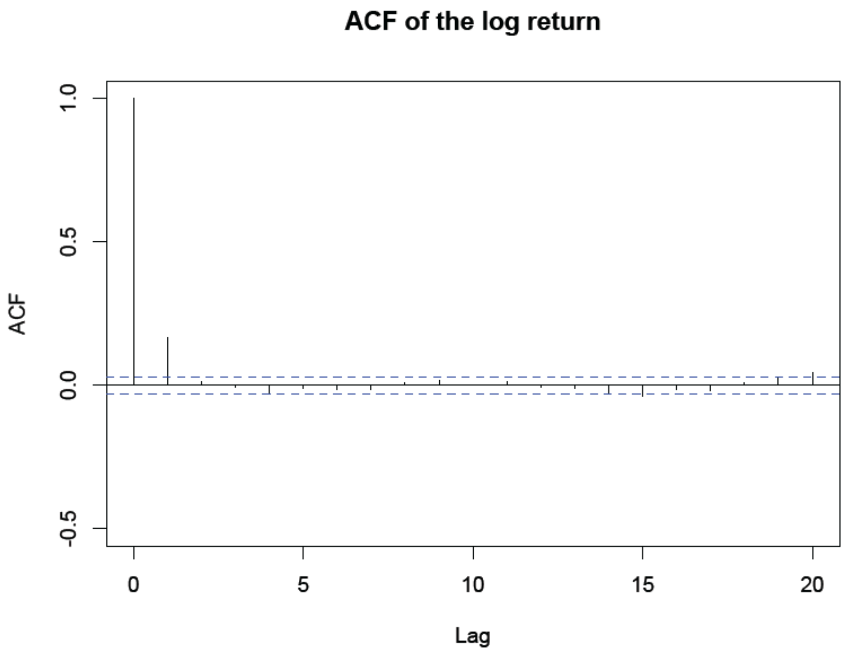

Fig 4. ACF of log return

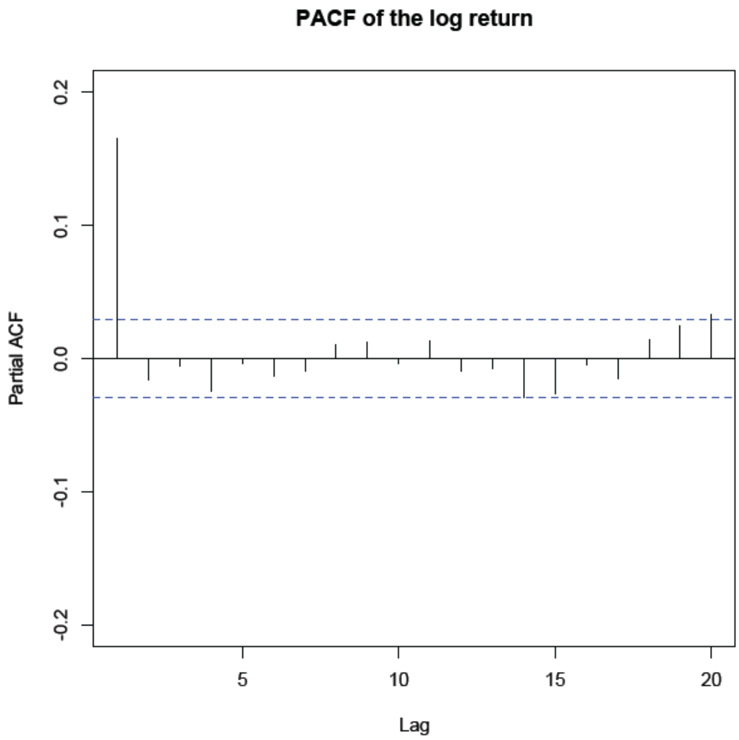

Fig 5. PACF of log return

A similar argument can be made for the PACF plot of the series exhibited in fig 5. Here, the lags are well contained with a little spill at lag 20.

From the two plots above, it may be inferred that an ARMA or ARIMA model would be a moving average model as the graphs displays its characteristics. Nevertheless, we apply other techniques that may help us establish a model for the series. We do this by constructing a matrix of models (with orders up to 4x4) and we choose the pair with the lowest Akaike information criterion (AIC) score. The residuals are also tested using the Box-Ljung test. This is to verify the residuals are white noise which is the desired objective. The hypothesis for this test is formulated as follows:

$$H_0: R \sim WN(0,\sigma^2) \text{ vs } H_a: R \nsim WN(0,\sigma^2)$$

where R is the residuals

σ^2 is the varainace

Under this test, H_0 is the null hypothesis and H_a is the alternative hypothesis.

ARIMA ANALYSIS

Tables 1-3 in the appendix shows the output of ARIMA models with orders *p* and *q* values ranging from zero to four. In table 1 we realize that the minimum AIC value is -3.5779.39 and belongs to the model ARIMA (0, 0, 1). The standard error at the specified *p* and *q* values also exhibits a minimum gradient to higher order. The box test for this model passed the random test.

An estimation of the model can be seen below in table 2. The standard error and the residual variance for our model is quite low (0.0146 and 1.978E-05) . This would be due to the relatively small changes that occur with foreign exchange rates in general.

Estimation	MA1	Mean
Coefficient	0.1659	0
Standard Error	0.0146	0.0001

Table 2 Estimated values for the ARIMA(0,0,1) model

ARIMA(0,0,1) Model:

$$X_t = Z_t + \theta Z_{t-1} \quad , \text{ where } \quad Z_t \sim WN(0,\sigma^2)$$

$$X_t = Z_t + 0.1659 Z_{t-1} \quad , Z_t \sim WN(0,1.978E{-}05)$$

Fig 6. Residuals plot over time

The residual of this model is shown below in fig.6. It can be observed that residuals have a value of zero but displays a non-constant spread especially during 2006 to 2012.

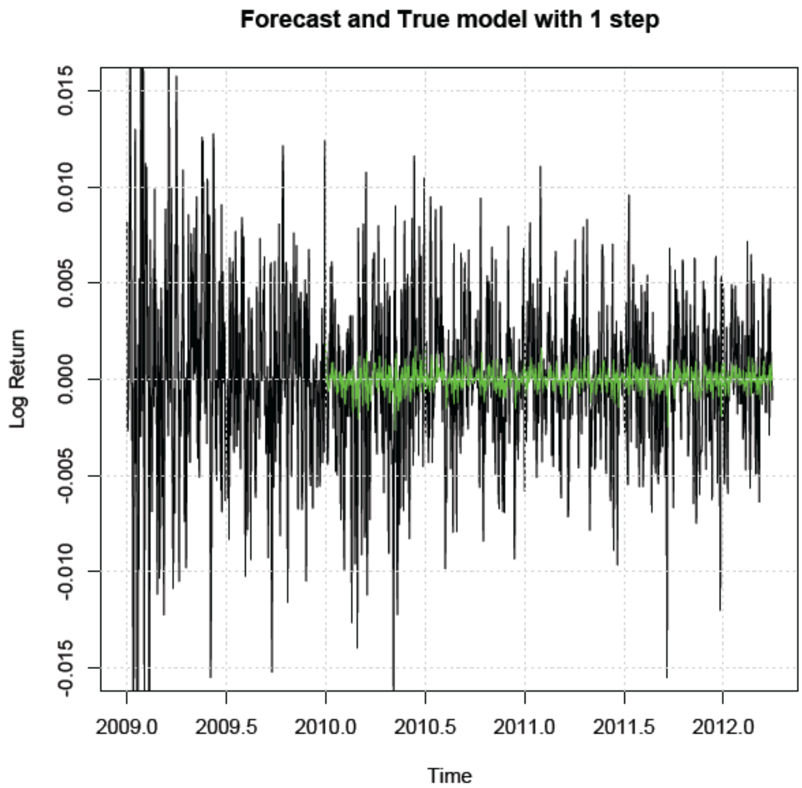

Fig 7. 1-step prediction of ARIMA(0,0,1) model

With our current model ARIMA(0,0,1) we tried to do 1-step prediction for the period of 2001-to 2012 to validate our model. This is shown Fig 7. The predicted values can be seen in the lighter color and it is an under prediction compared to the back drop true data (true returns). It is obvious that these prediction are not good enough. Due to the presence of a non-constant spread, we proceed to investigate the volatility of the time series as it is the most important factor for portfolio risk management.

GARCH ANALYSIS

GARCH (Generalized Autoregressive Conditional Heteroskedasticity) processes is a process where the conditional mean is constant but the conditional variance is non-constant and hence an uncorrelated but dependent process. The dependence of the conditional variance on the past causes the process to be dependent.

Conducting many GARCH models to fit out, we concluded that ARMA (1,0)-GARCH(1,2) was the best as it had the lowest AIC value of -8.162937 (see Tables 4 in the appendix).

ARMA (1,0)

An Autoregressive process X_t of order (lag) 1, AR (1), is defined as

$$X_t = c + \varphi X_{t-1} + Z_t, \qquad \text{where} \quad Z_t \sim WN(0,\sigma^2)$$

GARCH (1,2)

A Generalized Autoregressive Conditional Heteroskedasticity process of X_t order (1,2), GARCH(1,2) , is defined as

$$X_t = \sigma_t Z_t \quad ,\text{where} \quad Z_t \sim WN(0,\sigma^2)$$
$$\sigma^2 = \omega + \alpha X^2_{t-1} + \beta_1\sigma^2_{t-1} + \beta_2\sigma^2_{t-1}$$

ARMA (1,0)-GARCH(1,2)

A generalization of the GARCH(1,2) model, the ARMA(1,0)-GARCH(1,1) process, is defined

$$X_t = \sigma_t Z_t \quad ,\text{where} \quad Z_t \sim WN(0,\sigma^2)$$
$$\mu_t = \varphi X_{t-1}$$
$$\sigma^2_t = \omega + \alpha (X_{t-1} - \mu_{t-1})^2 + \beta_1\sigma^2_{t-1} + \beta_2\sigma^2_{t-1}$$

Where

X_t is the log return
Z_t is the shock or innovation
σ^2_t is the volatility
μ_t is the mean
and the coefficients and constant
$\alpha \geq 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$,
$(\alpha + \beta_1) < 1$ and $\beta_2 < 1$,
$\omega \geq 0$

Estimating the coefficients of our model yields the results in table 3. It is noticeable that the mean μ is not significant. Hence, we can drop it and the modified model is represented below.

ARMA(1,0)-GARCH(1,2) process, is defined

$$X_t = \sigma_t Z_t, \quad \text{where} \quad Z_t \sim WN(0,\sigma 2)$$
$$\mu_t = 0.1653\ X_{t-1}$$
$$\sigma^2_t = 0.05088\ X_{t-1}^2 + 0.154\ \sigma^2_{t-1} + 0.7888\ \sigma^2_{t-2}$$

Parameters	Estimates	Standard Error	t value	p-value
μ	2.46E-05	5.68E-05	0.433	0.66513
φ	0.16530	1.59E-02	10.412	< 2e-16
ω	1.20E-07	4.13E-08	2.903	0.00369
α	0.05088	5.48E-03	9.286	< 2e-16
β_1	0.15400	5.70E-02	2.703	0.00686
β_2	0.78880	5.65E-02	13.962	< 2e-16

Table 3 Estimated values for the ARIMA(1,0,0)-GARCH(1,2) model

The residuals and the standard residuals were also tested for significance using Ljung-Box Test. The squared of the residuals and the standard residuals were also accounted for (see table 5 in appendix).

The results shows that the residuals and the standard residuals were non-significant but their squared values are. This is evident in the residual plots in fig 8. Looking at the ACF of the squared standard residuals (lower right of fig 8), we can say the spikes out of the confidence region are moderate. These are due to small autocorrelations that should not be of practical importance and therefore can be ignored. Also, the plot of the standard residuals turn to have a constant spread compared to the actual residuals.

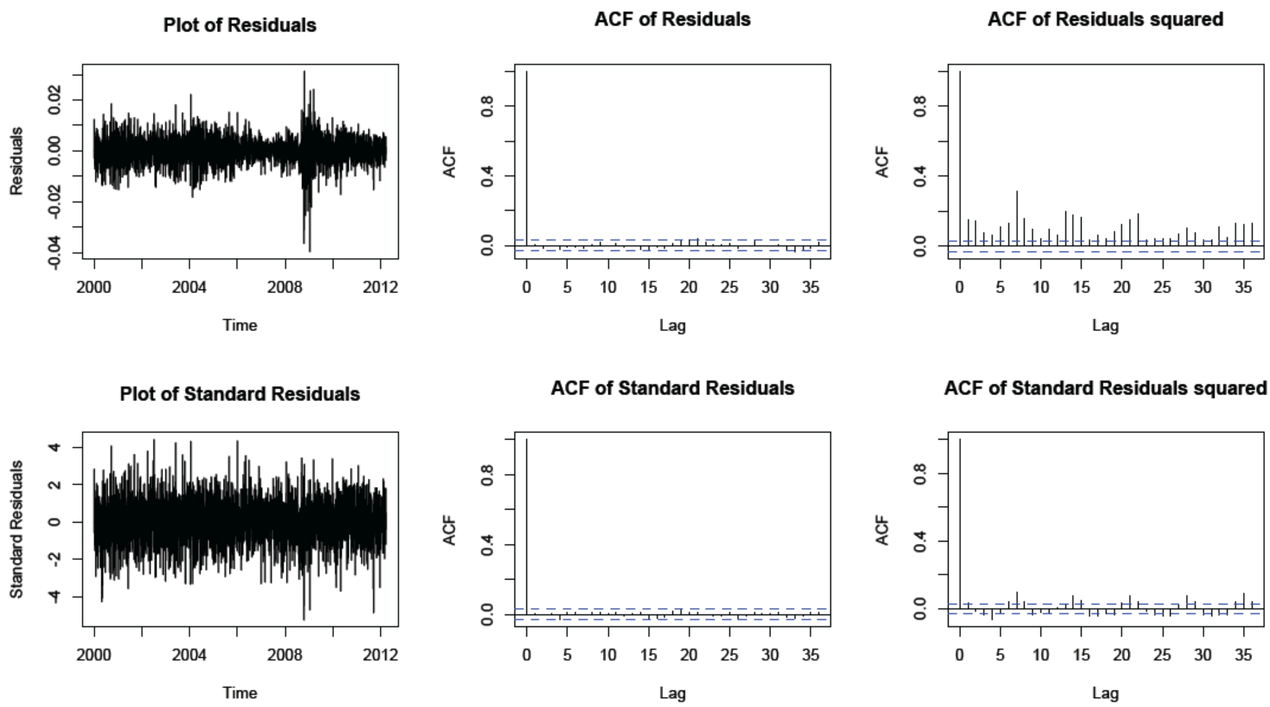

Fig 8. Residuals plots:

Plotting the volatility against time (fig 9), we find major turbulence around the period of the economic recession (2008 -2009).

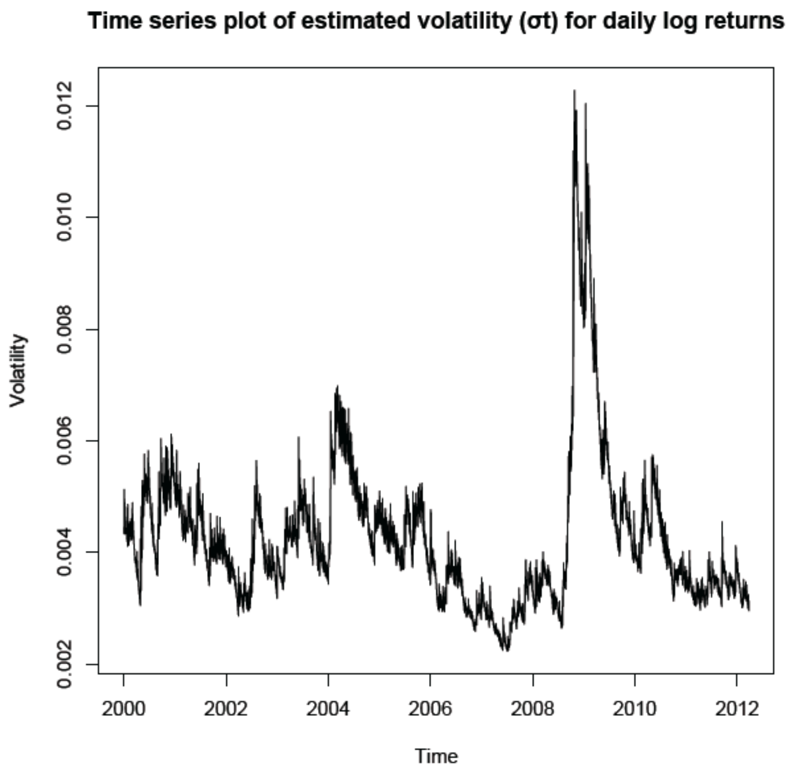

Fig 9. Volatility plot over time

A volatility prediction made between the period of 2010 to 2012 to validate our GARCH model indicates a better fit to the actual volatility as seen below. The actual plot is shown by the black line while the predicted values are represented by the lighter color.

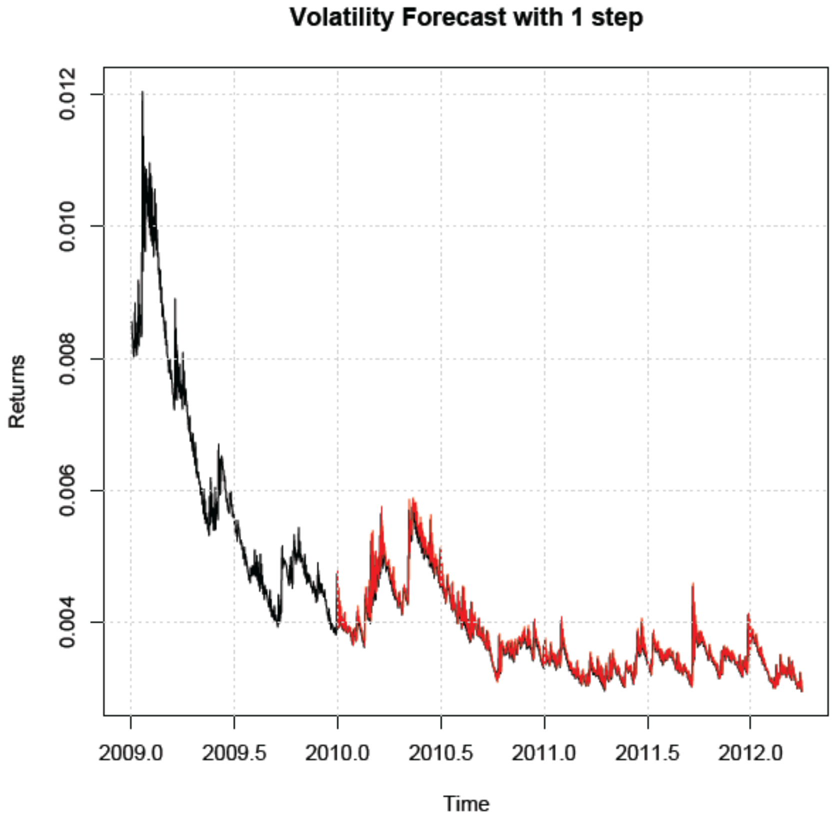

Fig 10. Forecasting volatility from 2010 to 2012

INTERPRETATION

To apply the ARMA modeling technique to a time series data, the data should be stationary. The logarithm of returns of the exchange rates are stationary, as seen in the ACF plots, thus, the integration part of the ARIMA process is zero.

Volatility in the data was fairly homogeneous from 2000 to 2006 with 2008 to 2012 being inconsistent. This condition makes fitting ARMA to financial data quite difficult. This was evident in the model orders - MA(1). The ARMA models obtained were not credible and therefore producing poor prediction outcomes.

Due to the changing variance observed in the series from 2007 to 2010, it was imperative that we analyze the variance of the series using ARMA(0,1)-GARCH(1,2). This resulted in a better prediction of the volatility.

CONCLUSION

Time series analysis and modeling is a very popular technique in mathematics and statistics used to explore the hidden details in time dependent data. ARIMA (ARMA) modeling is one of the basic time series methods employed in practice. In this study, we examined the foreign exchange rate between the GBP and the USD. Due the nature of the data, the logarithms of the returns of the rates are used in the analysis instead of the actual data. This is due to the favorable statistical properties of the logarithm of the returns provides for analysis.

As noted previously, ARIMA (ARMA) modeling fails to effectively capture the process being followed and subsequent forecasting chosen observations for validation. An application of an alternative modeling technique (ARCH/GARCH) is used be to analyze the volatility in the series and turn out to be of significant analysis due to the fact that it provides bankers and traders information about the "value at risk" for a given portfolio.

APPENDIX

p,q	0	1	2	3	4
0	-35656.95	**-35779.39**	-35778.08	-35776.1	-35776.46
1	-35778.99	-35778.07	-35776.08	-35774.08	-35774.59
2	-35778.13	-35776.08	-35774.11	-35772.07	-35772.61
3	-35776.26	-35774.18	-35774.57	-35773.46	-35770.61
4	-35776.87	-35774.8	-35772.82	-35771.44	-35769.32

Table 1 AIC values for estimated ARIMA(p,0,q)

p,q	0	1	2	3	4
0	2.0333E-05	**1.9776E-05**	1.9773E-05	1.9773E-05	1.9762E-05
1	1.9778E-05	1.9773E-05	1.9773E-05	1.9773E-05	1.9762E-05
2	1.9773E-05	1.9773E-05	1.9773E-05	1.9773E-05	1.9762E-05
3	1.9772E-05	1.9772E-05	1.9762E-05	1.9758E-05	1.9762E-05
4	1.9761E-05	1.9761E-05	1.9761E-05	1.9758E-05	1.9759E-05

Table 2 Standard error values for estimated ARIMA(p,0,q)

p,q	0	1	2	3	4
0	FALSE	**TRUE**	TRUE	TRUE	TRUE
1	TRUE	TRUE	TRUE	TRUE	TRUE
2	TRUE	TRUE	TRUE	TRUE	TRUE
3	TRUE	TRUE	TRUE	TRUE	TRUE
4	TRUE	TRUE	TRUE	TRUE	TRUE

Table 3 Box-Test for estimated ARIMA(p,0,q) with p-value > 0.01 as True

Model	AIC
GARCH(1,0)	-8.029915
GARCH(2,0)	-8.036744
GARCH(3,0)	-8.038956
GARCH(1,1)	-8.134582
GARCH(2,1)	-8.134145
GARCH(1,2)	-8.139603
GARCH(2,2)	-8.139157
GARCH(3,2)	-8.138769
ARMA(1,0)/GARCH(1,0)	-8.04498
ARMA(1,0)/GARCH(2,0)	-8.055509
ARMA(1,0)/GARCH(3,0)	-8.053061
ARMA(1,0)/GARCH(1,1)	-8.158708
ARMA(1,0)/GARCH(2,1)	-8.158275
ARMA(1,0)/GARCH(1,2)	**-8.162937**
ARMA(1,0)/GARCH(2,2)	-8.16249
ARMA(1,0)/GARCH(3,2)	-8.162107

Table 4 Estimation of GARCH model using AIC

Test	Data	Lags	Statistic	p-Value
Ljung-Box Test	R	Q(10)	6.027875	0.8129159
	R	Q(15)	10.56042	0.7830879
	R	Q(20)	18.80854	0.5343004
	R^2	Q(10)	103.3104	0
	R^2	Q(15)	145.0879	0
	R^2	Q(20)	175.7589	0
Arch Test	R	TR^2	91.06061	3.08E-14

Table 5 Standard residual tests for GARCH model

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